U.G. 5th Semester Examination - 2020

MATHEMATICS

[HONOURS]

Discipline Specific Elective (DSE)

Course Code : MATH-H-DSE-T-2B

(Differential Geometry)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 2×10=20

   a) Define osculating plane and write its equation in vector notation.

   b) What is involute of a curve? What are the involutes of a circular helix?

   c) Find the first fundamental form of a surface given by $x^1 = u,\ x^2 = v,\ x^3 = f(u,v)$.

   d) On the surface of revolution $x = u\cos\varphi,\ y = u\sin\varphi, z = f(u)$ what are the parametric curves?

   e) Define developable surface with an example.

[Turn over]

   f) Find the equation of the curve which is the intersection of the cylinders $F: y = x^2$ and $G: z = x^3$.

   g) Find the necessary and sufficient condition for orthogonality of the parametric curves on a surface.

   h) Find the unit normal vector of the surface $\vec{r} = (a\cos u, a\sin u, bv)$ a, b are constants.

   i) What is the integral curvature of a sphere?

   j) Show that a space curve is a plane curve if and only if its torsion is zero.

   k) Define Bertrand's curves with an example.

   l) If curvature of a plane curve is constant then prove that it is a circle.

   m) State Gauss-Bonnet theorem for surface.

   n) What is geodesic? Give an example.

   o) Define mean curvature of a surface. If mean curvature becomes 380, than what is its geometrical significance?

2. Answer any **four** questions: 5×4=20

   a) Prove that for a helix the ratio of curvature and torsion is constant.

b) Find the curvature and torsion of a space curve $\vec{r} = (a\cos t, a\sin t, bt)$, a, b are constants.

c) If a space curve is given by $\vec{r}(t)$, then prove that its curvature $k = \frac{|\dot{\vec{r}} \times \ddot{\vec{r}}|}{|\dot{\vec{r}}|^3}$.

d) Find the second fundamental form for a surface $\vec{r} = (u\cos v, u\sin v, cv)$, c being constant.

e) Determine whether the surface with the metric $ds^2 = v^2(du)^2 + u^2(dv)^2$ is developable or not.

f) Calculate the Gaussian curvature for a surface with metric $ds^2 = (du)^2 + \lambda^2(dv)^2$, where λ is a function of u and v.

3. Answer any **two** questions: 10×2=20

a) State and prove Mensnier's theorem for surface. Hence prove that $(k_g)^2 + (k_n)^2 = k^2$ where k_g is geodesic curvature of the curve on surface, k_n is normal curvature and k being the curvature of the curve. 6+4

b) Establish Serret-Frenet formulae for a space curve. 10

c) State and prove Rodrigue's formula. 10

d) Find the mean curvature of right helicoid $\vec{r} = (u\cos v, u\sin v, cv)$ and explain its geometrical significance. 8+2